Paper presented at the Earli 2007 conference,
Budapest, Hungary, August 28 - September 1, 2007

The influence of representational format on learner-generated domain representations and mathematical achievement

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Abstract

The purpose of this study was (a) to assess the effects of learner-generated domain representations on understanding combinatorics and probability theory and (b) to investigate the effects of the format (graphical, arithmetical, or textual) in which learners create their domain representation. A pretest-posttest design was applied in which four conditions were compared: three experimental conditions in which learners constructed a personal representation of the domain in a graphical, arithmetical, or textual format, and a control condition in which learners did not have to construct a representation. It was found that the construction of a domain representation significantly increases learning outcomes. Furthermore, it was found that the format in which learners express their knowledge does not directly affect learning outcomes or the quality of the created domain representations. However, the arithmetical format prevents most learners from engaging and succeeding in externalizing their knowledge.

Introduction

The domain of combinatorics and probability theory is hard to grasp for many learners. Some of the reasons for this have to do with mathematics or science domains in general but others have to do with specific characteristics of the domain of combinatorics.

One of the more general reasons for students' problems with science and mathematics problems is that novices often have a tendency to focus on superficial details rather than on understanding the principles and rules underlying a science or mathematics domain (Chi, Feltovich, & Glaser, 1981; de Jong & Ferguson-Hessler, 1986; Reiser, 2004). Science and mathematics problems require learners to go beyond the superficial details in order to recognize the concepts and structures that underlie the problem and to decide which operations need to be performed to solve it (e.g., Fuchs et al., 2004). In the case of probability instruction, the approach that needs to be taken to solve a problem is very dependent on the correct classification of the problem (Lipson, Kokonis, & Francis, 2003).

A second reason for students' difficulties is that the abstract and formal nature of often used arithmetical representations does not show the

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underlying principles or concepts as explicitly as pictorial and textual representations. Most learners tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations need to be performed on adjacent numbers, rather than reflections of principles and concepts underlying these procedures (Atkinson, Catrambone, & Merrill, 2003; Cheng, 1999; Greenes, 1995; Nathan, Kintsch, & Young, 1992; Niemi, 1996; Ohlsson & Rees, 1991). Therefore, they easily lose sight of the meaning of their actions. In this case, processing formal notations becomes an end in itself (Cheng, 1999). Learning of arithmetical procedures without conceptual understanding tends to be error prone, easily forgotten, and not readily transferable (Ohlsson & Rees, 1991).

Third, the formal, abstract way in which subject matter is represented makes it hard for learners to relate the subject matter to everyday life experiences. Fuson, Kalchman, and Bransford (2005) argue that the knowledge learners bring into the classroom is often put aside in mathematics instruction and replaced by procedures that disconnect problem solving from meaning making. In the case of probability and combinatorics the integration of theory and everyday life experience is particularly important, because probability ideas often appear to conflict with students' experiences and how they view the world (Garfield & Ahlgren, 1988; Kapadia, 1985). The conflicts arise because probabilities do not always fit learners' conceptions and intuitions (e.g., Batanero & Sanchez, 2005; Fischbein, 1975; Greer, 2001). An example of a misconception is the gambler's fallacy, that is, the belief that the outcome of a random event can be affected by (and therefore predicted from) the outcomes of previous events.

These reasons are by no means exhaustive, but summarize some of the main problems encountered in the instruction of combinatorics and probability theory. Learners end up with a knowledge base that is biased towards procedural knowledge, which means that they know how to perform arithmetical operations but hardly understand the underlying principles, or the conditions under which the procedures are applicable. Moreover, they fail to relate their knowledge to everyday life situations. Instructional approaches therefore need to concentrate on ways to focus the learners' attention to the principles and rules underlying the domain rather than to superficial details and operations, while using representational formats that are most beneficial for understanding, preferably in a way that relates theoretical mathematical knowledge to everyday life experiences.

Using externalization activities to foster understanding

Gaining a full understanding of a domain requires learners to combine conceptual, procedural, and situational knowledge into meaningful schemata. These schemata can be acquired by performing cognitive activities such as selecting, organizing, and integrating information (Mayer, 2003, 2004; Shuell, 1986, 1988; Sternberg, 1984). Selecting involves recognizing which information is relevant and which is not. Organizing involves combining pieces of information into a coherent and internally connected structure (e.g., a mental representation). Integrating refers to relating newly acquired knowledge to already existing knowledge structures (prior knowledge). Many learners do not engage in these cognitive activities, unless prompted to do so (Pressley et al., 1992). A promising way of prompting is to stimulate learners to externalize their domain knowledge. Cox (1999), for example, argues that the externalization of knowledge may elicit self-explanation effects and that the process of externalization consists of dynamic iterations and interactions between external models and mental models and therefore helps learners to refine and disambiguate their knowledge of the domain.

There are many ways to stimulate learners to externalize their domain knowledge. Frequently used instructions include asking the learners to write summaries (Foos, 1995; Hidi & Anderson, 1986; Spurlin, Dansereau, O'Donnell, & Brooks, 1988), or to create drawings (Van Meter, Aleksic, Schwartz, & Garner, 2006; Van Meter & Garner, 2005) of the subject matter. A less well-known technique is letting learners build their own runnable computer model (Löhner, Van Joolingen, & Savelsbergh, 2003; Manlove, Lazonder, & de Jong, 2006; van Joolingen, de Jong, Lazonder, Savelsbergh, & Manlove, 2005). In this approach, learners construct a dynamic simulation of their own mental model. They can observe the model they created and draw conclusions based on the model output. Another example of externalization is asking the learners to construct a concept map during or after learning (Gijlers, 2005; Nesbit & Adesope, 2006; Novak, 1990, 2002; O'Donnell, Dansereau, & Hall, 2002). A learner-generated concept map can give insight into which concepts are thought to be relevant or important and how the learner thinks the concepts are related to each other, and thus shows where gaps or misconceptions occur.

The type of externalization is related to the representational format in which learners are expected to express their knowledge. Representational format is known to play a critical role in learning and understanding (Ainsworth & Loizou, 2003; Cheng, 1999; Zhang, 1997). The properties of representations are assumed to influence which information is attended to and how people tend to organize, interpret, and remember the information (e.g., Larkin & Simon, 1987). With regard to learner-generated externalizations of knowledge in the domain of combinatorics and probability theory, it has been found that learners avoid using conventional ways of representing the probability of events (i.e., using ratios or odds, or formal numerical probabilities) and prefer to use alternative forms of representation, ranging from textual statements to conventional numerical representations (Tarr & Lannin, 2005). This finding indicates that all formats may not equally suitable for learners trying to express their knowledge.

Research questions

The aim of the current study was to find out whether externalizing knowledge leads in general to better understanding in the domain of combinatorics and probability, and in particular, whether the representational format in which learners externalize their knowledge has a differential effect on understanding in this domain. Three formats in which learners could externalize their knowledge were compared: (a) a graphical format (i.e., concept maps), (b) an arithmetical format, and (c) a textual format. Externalization is assumed to focus the learners' attention more on conceptual and situational aspects of the domain, resulting in a knowledge base that contains conceptual knowledge, procedural knowledge, and situational knowledge instead of a one-sided focus on procedural knowledge. Externalization in the graphical format is hypothesized to stimulate conceptual knowledge in particular, because of the primary focus on the identification of concepts and their mutual relationships (Nesbit & Adesope, 2006). Although expressing knowledge in an arithmetical format is supposed to draw the learners' attention primarily to operational aspects, externalizing is still thought to improve conceptual and situational knowledge as compared to learners who do not externalize at all. Externalizations of a textual nature are assumed to direct the learners' attention to conceptual and situational aspects, although the conceptual issues might not be as strongly stressed as for learners who construct a concept map.

Method

Participants

In total, 133 third-grade pre-university education students, 65 boys and 62 girls (six participants did not indicate their gender), participated. The average age of the participants was 14.63 years (SD =.62). The participants had no prior knowledge of combinatorics and probability theory. They attended the experiment during regular school time; therefore, participation was obligatory. Participants received a mark based on their post-test performance.

Design

The experiment employed a between-subjects design with the representational format (graphical, arithmetical, textual) in which learners had to express their knowledge as the independent variable. Participants in a control condition were provided with the same learning environment, assignments, and tests as participants in the experimental conditions. The only difference was that participants in the control condition were not required to externalize their knowledge. Participants were assigned randomly to conditions. Twelve participants were excluded from the analyses; seven of them did not attend one or more experimental sessions, and five were excluded because their post-test scores deviated more than 2 SDs from the mean scores within their condition. The distribution of the remaining 121 participants across conditions is displayed in Table 1.

Table 1
Number of participants per condition

	Condition			
	Graphical	Arithmetical	Textual	Control
Number of participants	31	28	30	32

Domain

The domain of instruction was combinatorics and probability theory. An example of a problem in this domain is: what is the probability that a thief will guess the 4-digit PIN-code of your credit card correctly in one go? The essence of combinatorics is determining how many different combinations can be made with a certain set or subset of elements. In order to determine the number of possible combinations, one also needs to know 1) whether elements may occur repeatedly in a combination (replacement) and 2) whether the order of elements in a combination is of interest (order). On basis of these two criteria, four so-called problem categories can be distinguished (for an overview, see Figure 1).

		ORDER IMPORTANT?	
		Yes	**No**
REPLACEMENT?	**No**	Category 1: No replacement; Order important	Category 2: No replacement; Order not important
	Yes	Category 3: Replacement; Order important	Category 4: Replacement; Order not important

Figure 1. Problem categories within the domain of combinatorics

The PIN-code example matches category 3 (replacement; order important). When the number of possible combinations is known, the probability that one or more combinations will occur in a random experiment can be determined.

Externalization tools

Participants in the experimental conditions were asked to construct a representation of the domain that would be meaningful to themselves and a fictitious fellow learner. This representation was to reflect the relevant principles underlying the domain, the variables playing a role in the domain, and their mutual relationships. Learners had to create their representations on an electronic on-screen externalization tool. There were three types of externalization tool, one for each experimental condition: (a) a graphical externalization tool (i.e., a concept mapping tool), (b) an arithmetical externalization tool, and (c) a textual externalization tool.

In the graphical externalization tool, learners were supposed to create concept maps of the domain. Learners could draw circles representing domain concepts and variables. Keywords could be entered in the circles. The circles could be connected to each other by arrows indicating relations between concepts and variables. The nature of these relations could be specified by attaching labels to the arrows. In the arithmetical externalization tool, learners could use variable names (N, K, and P), numerical data, and mathematical operators (division signs, equation signs, multiplication signs, and so on) in order to express their knowledge. Finally, the textual externalization tool resembled simple word processing software, allowing textual and numerical input. The contents of the externalization tools were stored automatically.

Learning environment

The instructional approach used in this study is based on inquiry learning (de Jong, 2005, 2006). Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. By manipulating the input variables and observing the resulting changes in output values the learner is enabled to induce the concepts and principles underlying the model (de Jong & van Joolingen, 1998).

The learning environment used in the current study, called Probe-XMT, was created with SIMQUEST authoring software (van Joolingen & de Jong, 2003). Probe-XMT consisted of five sections. Four of these sections were devoted to each of the four problem categories within the domain of combinatorics. The fifth section aimed at integrating these four problem categories. Each section used a different cover story, that is, an everyday life example of a situation in which combinatorics and probability played a role. Each cover story exemplified the problem category treated in that section. In the fifth (integration) section, the cover story applied to all problem categories.

Each of the five sections contained a series of questions (both open-ended and multiple-choice items), all based on the cover story for that particular section. These questions involved determining which problem category matched the given cover story (situational knowledge), calculating the probability in a given situation (procedural knowledge), and selecting a description that matched the relation between variables most accurately (conceptual knowledge). In the case of the multiple-choice items, the learners received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Learners then had the opportunity to select another answer. In the case of the open-ended questions, learners received the correct answer after completing and closing the question.

Most of the questions were accompanied by simulations that could be used to explore the relations between variables within the problem category. In the simulations, learners could manipulate variables and observe the effects of their manipulations on other variables. The simulations used a combination of textual and arithmetical representations.

The learning environment automatically registered user actions. User actions that were logged included measures like user path through the learning environment (which parts of the learning environment were opened, when, for how long, and in what sequence) and the number and nature of manipulations carried out in the simulations (how many experiments were carried out and the input values of each experiment).

Knowledge measures

Two knowledge tests were used in this experiment,: a pre-test and a post-test. These tests contained 12 and 26 items respectively. An overview of the test items and underlying knowledge types is presented in Table 2.

Table 2

Overview of test items

Knowledge type	Description	Number and type of items
Pretest		
Conceptual	Items about relations between variables	4 mc
Procedural	Solving combinatorial problems	4 open and 4 mc
Post-test		
Conceptual	Items about relations between variables	13 mc
Procedural	Solving combinatorial problems	9 open
Situational	Analyzing, identifying, and classifying problems	4 mc

In cases where learners had to calculate the outcome of an item, a calculator was provided on-screen. Both tests were administered via the internet. The questions were presented screen-by-screen without the possibility of skipping questions or turning back to previous questions. Learner responses on both multiple-choice and open-ended items were collected and recorded electronically.

Procedure

The experiment was carried out in a real school setting in three sessions, each separated by a one-week interval. Learners worked individually and they were told that they could work at their own pace. The first session started with some background information with regard to the experiment (general purpose of the research, the domain of interest, learning goals, etcetera). This was followed by the pre-test. It was announced that the post-test would contain more items of greater difficulty than the pre-test, but that the pre-test items nonetheless would give an indication of what kind of items to expect on the post-test. At the end of the pre-test the learners received a printed introductory text in which the domain was introduced. The duration of the first session was limited to 50 minutes. During the last 15 minutes of the session, the experimenter demonstrated the use of the learning environment and the externalization tools.

During the second session, learners were working with the learning environment, whereby learners in the experimental conditions had to construct a domain representation while working with the learning environment. The duration of this session was set at 70 minutes. Despite the possibility of following a non-linear path through the learning environment, learners were advised to keep to the order of sections because they build upon each other.

The third session was set at 50 minutes. First, learners were allowed to use the learning environment for 10 minutes in order to refresh their memories with regard to the domain. Then all learners had to close their domain representations and learning environments, and had to complete the post-test. When learners finished the test they were allowed to leave the classroom.

Data preparation

The domain representations constructed by the learners were scored by means of a scoring protocol. This protocol revolved around the principle that scoring of the domain representation should not be biased by the representational format of the externalization tool, that is, all types of representations should be scored on the basis of exactly the same criteria. The protocol was used to assess the extent to which domain representations reflected the concepts of replacement and order, presented calculations, referred to the concept of probability, indicated the effect of size of (sub)sets on probability, and the effects of replacement and order on probability. The maximum number of points that could be assigned on the basis of the protocol was 8 points. The scoring protocol is displayed in Appendix I.

Results

Knowledge measures

Two measures of knowledge were obtained: prior knowledge (pre-test score), and post-test score. The reliability, Cronbach's α, was α = .56 for the pre-test and α = .79 for the post-test. The scores on the knowledge measures are displayed in Table 3.

Table 3
Knowledge measures

	Condition							
	Graphical (n=31)		Arithmetical (n=28)		Textual (n=30)		Control (n=32)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Pre-test score	5.71	1.40	5.46	1.92	5.23	1.59	5.47	1.95
Post-test score	16.97	3.76	16.57	3.84	16.13	3.48	15.44	4.68

One-way analyses of variance (ANOVA) showed that there were no differences between conditions with regard to pre-test score, $F(3,120) = 0.39$, *ns*, and post-test score, $F(3,120) = 0.85$, *ns*. These results suggest that the externalization of knowledge and the representational format in which learners externalize do not affect learning outcomes. However, it turned out that not all participants in the experimental conditions externalized their knowledge. This issue will be addressed in the next section after, which a re-analysis of the data will be presented.

Externalizers vs non-externalizers

Not all learners who were required to externalize their knowledge did so, and some only entered nonsense input (i.e., input that was not domain related and that could not be considered a serious attempt to create a domain representation; e.g., a drawing of a smiling face). Therefore, a distinction will be made between the group of learners who did externalize their knowledge (henceforth called "externalizers"), and the group of learners who did not externalize although required to (henceforth called "non-externalizers"). The numbers of externalizers and non-externalizers for each condition are displayed in Table 4.

Table 4
Numbers of externalizers and non-externalizers

	Representational format		
	Graphical	Arithmetical	Textual
Total number of participants	31	28	30
Number of non-externalizers	15	23	16
Number of externalizers	16	5	14
Percentage of externalizers	51.6	17.9	46.7

The numbers of externalizers and non-externalizers differed significantly between conditions, $\chi^2(5, N = 89) = 11.25$, $p < .05$. In the Arithmetical condition there were significantly fewer externalizers than in both the Graphical condition, $\chi^2(1, N = 21) = 5.76$, $p < .05$, and the Textual condition, $\chi^2(1, N = 19) = 4.26$, $p < .05$.

Knowledge measures revisited

The scores on the knowledge measures were reanalyzed, this time taking into account the distinction between externalizers and non-externalizers. Before reanalyzing, the comparability of the different groups with regard to prior knowledge (pre-test score) was established (see Table 5).

Table 5
Prior knowledge measure

	Group					
	Externalizers (*n*=35)		Non-Externalizers (*n*=54)		Control (*n*=32)	
	M	*SD*	*M*	*SD*	*M*	*SD*
Pre-test score	5.74	1.54	5.30	1.68	5.47	1.95

A one-way ANOVA showed no difference between groups with regard to pre-test scores, $F(2,120) = 0.72$, *ns*.

The post-test scores, including the measures of the different knowledge types, were analyzed by applying one-way ANOVAs. The post-test measures for each group are displayed in Table 6.

Table 6
Post-test measures for each group

	Group					
	Externalizers (*n*=35)		Non-Externalizers (*n*=54)		Control (*n*=32)	
Knowledge type	*M*	*SD*	*M*	*SD*	*M*	*SD*
Conceptual knowledge	10.06	1.14	9.35	1.54	9.06	1.90
Procedural knowledge	4.77	2.20	3.54	2.23	3.75	2.57
Situational knowledge	3.46	0.85	2.56	1.33	2.63	1.36
Post-test overall score	18.29	3.07	15.44	3.62	15.44	4.68

With regard to the post-test overall score, a significant difference was found between groups, $F(2,118) = 7.01$, $p < .01$. A post-hoc LSD analysis showed that the externalizers outperformed both the non-externalizers ($p < .001$) and the control group ($p < .01$). The post-test distinguished between three types of knowledge. One-way ANOVA's revealed significant differences between conditions with regard to conceptual knowledge, $F(2,118) = 3.79$, $p < .05$, procedural knowledge, $F(2,118) = 3.19$, $p < .05$, and situational knowledge, $F(2,118) = 6.45$, $p < .01$. Post-hoc LSD analyses showed that with regard to conceptual knowledge the externalizers outperformed the non-externalizers ($p < .05$) and the control group ($p < .01$). With regard to procedural knowledge the externalizers outperformed the non-externalizers ($p < .05$) but not the control group. Finally, the average scores of externalizers on situational knowledge were also significantly higher compared to non-externalizers ($p < .001$) and the control group ($p < .01$).

In order to find the answer to our second research question (does the representational format in which learners externalize their knowledge have a differential effect on understanding?) we now focus on the externalization group. The post-test scores of externalizers using different representational formats are displayed in Table 7.

Table 7
Post-test measures of externalizers

	Representational format					
	Graphical (*n*=16)		Arithmetical (*n*=5)		Textual (*n*=14)	
Knowledge type	*M*	*SD*	*M*	*SD*	*M*	*SD*
Conceptual knowledge	10.06	1.12	10.20	0.84	10.00	1.30
Procedural knowledge	5.13	2.03	5.60	3.36	4.07	1.86
Situational knowledge	3.56	0.81	3.60	0.89	3.29	0.91
Post-test overall score	18.75	3.04	19.40	4.22	17.36	2.62

No differences were found between representational formats with regard to post-test measures. The $F(2,32)$ values for conceptual, procedural, situational, and post-test overall scores are 0.05, 1.30, 0.46, and 1.16 respectively. Therefore, the format in which learners expressed their knowledge did not influence post-test scores.

The quality of domain externalizations

The domain representations created by the externalizers show large differences. The quality of the domain representations is determined by the extent to which the externalization properly represents the domain and contains all relevant concepts and relations. The quality of the created domain representations was determined on the basis of the coding scheme presented earlier in this paper (see also Appendix I). The resulting quality scores are displayed in Table 8.

Table 8
Quality scores of learner-generated domain representations (externalizations)

	Representational format											
	Graphical				Arithmetical				Textual			
	M	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*
Score	2.38	1.20	1	5	3.40	1.67	2	6	2.57	1.40	1	5

A one-way ANOVA showed that there were no differences between representational formats with regard to quality scores, $F(2,34) = 1.10$, *ns*.

Discussion and Conclusion

The aim of the current study was to find out whether externalizing knowledge leads to better understanding of the domain and in particular whether representational format has a differential effect on this. The results show that the externalization of knowledge significantly enhances conceptual knowledge, procedural knowledge, situational knowledge and overall post-test performance. Therefore, externalization leads to better understanding of the principles that govern the domain and of the interrelations between units of knowledge in the domain (conceptual knowledge), enhanced ability to execute action sequences to solve problems (procedural knowledge), and to analyze, identify, and classify problems, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem (situational knowledge). Moreover, earners who externalized their knowledge did not differ from other learners with regard to time-on-task. Therefore, the effects cannot be attributed to the time the learners spent on their learning process. Externalization appears to be an effective method to enhance learning results, without requiring the investment of much time.

The second research question concerned the different representational formats in which learners could externalize their knowledge. It was hypothesized that externalization in the graphical format would lead to more conceptual knowledge, that externalization in the arithmetical format was supposed to draw the learners' attention primarily to operational aspects, although externalizing still was thought to improve conceptual and situational knowledge as compared to learners who do not externalize at all. Externalization in the textual format was assumed to lead to more conceptual and situational knowledge, although the focus on conceptual aspects would not be as marked as in the case of the graphical format. Results show that the hypotheses were not confirmed. No differential effects on post-test scores were found between representational formats. Furthermore, it turned out that the representational format in which learners created a domain representation did not affect the quality of these domain representations.

Although representational format did not have a direct effect on knowledge measures and quality of domain representations, it turned out that format did play an important, although indirect role. The format determined to a large extent the likelihood that learners engaged (and succeeded) in externalizing their knowledge. The graphical and textual formats appear to equally foster the externalization of knowledge. Having learners express their knowledge in an arithmetical format is not recommended, because this format prevented most learners from engaging and succeeding in externalizing their knowledge. The low number of learners who succeeded in constructing a domain representation by using the arithmetical format suggests that this format is not very helpful for externalizing knowledge. This finding corroborates the observation that learners apparently experience difficulties with using standard mathematical representations. If learners have the choice, they prefer to use other types of representations (Tarr & Lannin, 2005). Still, the finding that learners apparently consider the arithmetical format too difficult for externalizing their knowledge is interesting. In an earlier study, Kolloffel, de Jong, and Eysink (2005) found that in learning from (pre-constructed) representations, the best results were obtained with arithmetical representations. This suggests that arithmetical representations are very well suited for learning, but not for expressing knowledge.

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Appendix I

	REPRESENTED?	CONCEPT MAP	MATH EDITOR	TEXT EDITOR	PNT
A	The concept of "Replacement"	-Literally, or descriptive *Examples:* -*"Replacement" (Abstract)* -*"Category 1: without replacement; order important" (Abstract)* -*...[Runners, BK]... then you have to do 1/7 x 1/6 x 1/5 because each time there is one runner fewer" (Concrete)*	Two formulas or calculations in which "replacement" varies *Examples:* -*"(1/n) x (1/n) x (1/n) = P* *(1/n) x (1/(n-1)) x (1/(n-2))= P"* -*"1/5 x 1/4 x 1/3* *1/5 x 1/5 x 1/5"* -*"p = 1/10 x 1/10 x 1/10* *p = 1/5 x 1/4 x 1/3"*	-Literally, or descriptive *Examples:* -*"Replacement" (Abstract)* -*"Category 1: without replacement; order important" (Abstract)* -*"...If there are 7 runners, then the chance is 1 out of 7 (1/7), if that runner passes the finish, then there are 6 runners left, then there is a chance of 1 out of 6 (1/6), and so on. (Concrete)*	1
B	The concept of "Order"	-Literally, or descriptive *Examples:* -*"Order" (Abstract)* -*"Category 1: without replacement; order important" (Abstract)* -*"...If there are 7 runners and you predict the top 3 without specifying the positions of specific runners in the top 3..." (Concrete)*	Two formulas or calculations in which "order" varies *Examples:* -*"(1/n) x (1/n) x (1/n)* *(k/n) x ((k-1)/n) x ((k-2)/n)"* -*"1/5 x 1/4 x 1/3* *3/5 x 2/4 x 1/3"*	-Literally, or descriptive *Examples:* -*"Order" (Abstract)* -*"Category 1: without replacement; order important" (Abstract)* -*"...At a game of Bingo, order is not important" (Concrete)*	1
C	Calculation	-Formal, literally, descriptive, or a concrete calculation *Examples:* -*p = acceptable outcomes/ possible outcomes (Abstract)* - *1/5 x 1/4 x 1/3 (Procedural)* -*... when you also bet ont he order in which the marbles will be selected, your chance*	Formal (formula) or a concrete calculation *Examples:* -*"(1/n) x (1/n) x (1/n)"* -*"1/5 x 1/4 x 1/3"*	-Formal, literally, descriptive, or a concrete calculation *Examples:* -*p = acceptable outcomes/ possible outcomes (Abstract)* - *1/5 x 1/4 x 1/3 (Procedural)* -*... when you also bet ont he order in which the marbles will be selected, your chance*	1

		is: 1/5 and 1/4 is 1/20..." (Concrete)		*is: 1/5 and 1/4 is 1/20..." (Concrete)*	
D	Probability	-Literal reference to the term "probability"/p, or a description of the concept -Expression of a concrete probability (e.g. a fraction), but then it need to be made clear in the context (e.g. by a calculation) where the probability comes from *<u>Examples</u>:* -"*In order to calculate 'p' the chances need to be multiplied." (Abstract)* *-p= 1/5 x 1/4 x 1/3 (Procedural)* -"*...In that case [learner refers to a situation outlined earlier], the probability is 1/10" (Concrete)*	-Literal reference to the term "p" -Expression of the outcome of a calculation *<u>Examples</u>:* -"*p = (1/n) x (1/n) x (1/n)"* -"*p = 1/5 x 1/4 x 1/3"* -"*1/5 x 1/4 x 1/3 = 1/60"*	-Literal reference to the term "probability"/p, or a description of the concept -Expression of a concrete probability (e.g. a fraction), but then it need to be made clear in the context (e.g. by a calculation) where the probability comes from *<u>Examples</u>:* -"*In order to calculate 'p' the chances need to be multiplied." (Abstract)* *-p= 1/5 x 1/4 x 1/3 (Procedural)* -"*...In that case [learner refers to a situation outlined earlier], the probability is 1/10" (Concrete)*	1
E	Effect of *n* on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *k* needs to be constant) *<u>Examples</u>:* -"*fewer options = higher chance" (Abstract)* -"*If fewer runners attend the race, the chance your prediction is correct will increase" (Concrete)*	A formula or a series of calculations showing the effect (in the latter case, *k* needs to be constant) *<u>Examples</u>:* -"*(1/n) x (1/n) x (1/n)=1/n*^{3}*"* -"*1/5 x 1/4 x 1/3 = 1/60 1/6 x 1/5 x 1/4 = 1/120"*	-Descriptive or on basis of calculations showing the effect (in the latter case, *k* needs to be constant) *<u>Examples</u>:* -"*If the number of elements you can choose from increases, the chance will be smaller that you will select a specific element" (Abstract)* -"*If fewer runners attend the race, the chance your prediction is correct will increase" (Concrete)*	1
F	Effect of *k* on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* needs to be constant)	A formula or a series of calculations showing the effect (in the latter case, *k* needs to be constant)	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* needs to be constant)	1

		Examples: -*"with 1 choice →1/possible outcomes; with more choices →number of choices/possible outcomes" (Abstract)* -*"If you only predict who will win the race and not the top 3, then the chance is greater that your prediction will be correct" (Concrete)*	Examples: -"$(1/n) \times (1/n) = 1/n^2$ $(1/n) \times (1/n) \times (1/n) = 1/n^3$" -"$1/5 \times 1/4 = 1/20$ $1/5 \times 1/4 \times 1/3 = 1/60$"	Examples: -*"When your prediction is less elaborate, the probability that your prediction will be correct increases" (Abstract)* -*"If you only predict who will win the race and not the top 3, then the chance is greater that your prediction will be correct" (Concrete)*	
G	Effect of replacement on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) Examples: -*"If it is a matter of replacement, your chances will decrease" (Abstract)* -*"...if you have 10 different cell phones and you need to select one, your chance will be 1 out of 10, if you put the phone back your chance will be 1 out of 10 again, but if you leave it out your chance will increase that you will select the next phone as predicted" (Concrete)*	A series of formulas or calculations showing the effect, but the outcome (*p*) needs to be represented as well and *n* and *k* need to be constant Examples: -"$(1/n) \times (1/n) = 1/n^2$ $(1/n) \times (1/(n-1)) = 1/(n^2-n)$" -"$1/5 \times 1/4 \times 1/3 = 1/60$ $1/5 \times 1/5 \times 1/5 = 1/125$"	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) Examples: -*"If it is a matter of replacement, your chances will decrease" (Abstract)* -*"...if you have 10 different cell phones and you need to select one, your chance will be 1 out of 10, if you put the phone back your chance will be 1 out of 10 again, but if you leave it out your chance will increase that you will select the next phone as predicted" (Concrete)*	1
H	Effect of order on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) Examples: -*" If order is important, the chance your prediction will be right will decrease" (Abstract)* -*"...If there are 7 runners and you predict the top 3, then*	A series of formulas or calculations showing the effect, but the outcome (*p*) needs to be represented as well and *n* and *k* need to be constant Examples: -"$(1/n) \times (1/n) = 1/n^2$ $(k/n) \times ((k-1)/n) = (k^2-k)/n^2$" -"$1/5 \times 1/4 \times 1/3 = 1/60$ $3/5 \times 2/4 \times 1/3 = 6/60$"	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) Examples: -*" If order is important, the chance your prediction will be right will decrease" (Abstract)* -*"...If there are 7 runners and you predict the top 3, then*	1

		the probability is 1/7 x 1/6 x 1/5 = 1/210, but without specifying the positions of specific runners in the top 3 the probability is 3/7 x 2/6 x 1/5 = 6/210..." (Concrete)		*the probability is 1/7 x 1/6 x 1/5 = 1/210, but without specifying the positions of specific runners in the top 3 the probability is 3/7 x 2/6 x 1/5 = 6/210..." (Concrete)*	
				MAXIMUM NUMBER OF POINTS	8